What Is The Converse Of The Pythagorean Theorem?

You may already know the Pythagorean theorem, but let's briefly cover it. The Pythagorean theorem is a formula we use with right triangles to show the relationship between the legs of a right triangle and the hypotenuse. This is expressed with A^2+B^2=C^2, with a and b representing the legs and c representing the hypotenuse.

The Pythagorean theorem can be stated thus, " If a triangle is a right triangle, then the relationship between the legs and the hypotenuse is a^2 + b^2=c^2 ".

The converse of a theorem is rewording a conditional statement. Conditional statements are worded as "If p, then q", it's converse would be "If q, then p". If both a conditional statement and its converse are true, then the statement can be reworded as a biconditional. Biconditionals are worded as "p if and only if q".

With this knowledge, let us analyze our previous conditional statement:

"If a triangle is a right triangle, then the relationship between the legs and the hypotenuse is A^2 + B^2 = C^2".

Let's now formulate the converse of the Pythagorean theorem.

"If the relationship between the legs and the hypotenuse of a triangle is A^2 + B^2 = C^2, then the triangle is a right triangle."

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Ask yourself, "***Is this converse statement strictly and necessarily true?***". In this case the answer is yes! Right triangles always have this relationship between the legs and the hypotenuse and likewise when a triangle' sides have this relationship it must necessarily be true.

If both a conditional statement and its converse are true, then the statements can also be expressed as a biconditional.

"p if and only if q"

in the case of this example

"A triangle is a right triangle if and only if the relationship between the legs is A^2 +B^2 = C^2

Since the converse of the Pythagorean theorem is true, we can use it when solving proofs. When you see that the sum of the square of each leg is equal to the square of the hypotenuse, we can therefore conclude that the given triangle is a right triangle.

Let's consider an example:

Suppose that we must prove that a given triangle is a right triangle.

No angle measures are given, but the sides are given as follows,

3,4,5

Remember that the longest side is the hypotenuse, c. Therefore c = 5.

The legs are 3 and 4, which we can assign to a and b respectively.

If we square each leg and then add them to equal the square of the hypotenuse we get

$a^2 + b^2=c^2$

$3^2 + 4^2=5^2$

$9 + 16 = 25$

$25=25$

Since the Pythagorean theorem works on this triangle, we can therefore conclude that the triangle is a right triangle.

The reason given would be: converse of the Pythagorean theorem.

Keep in mind that although the converse of the Pythagorean theorem is true, converses of other theorem and proofs may not be.